Real-time Fuzzy Digital Filter: Basic Concepts.

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Abstract. This work presents an analysis of fuzzy logic (to see: [1], [3], [8], [9], [10], [11]) applied to digital filters (to see: [2], [4], [8] and [12]) and real-time systems (to see: [15] and references in it), for to do a concept of real time fuzzy digital filters. We will describe the basic characteristics of operation and meaning of this kind of filters in order to know how the signals will be classified by the filter (to see: [12] and [15]), we resolve how to construct and characterize the membership function [3] and the knowledge base in a probabilistic manner [12], making a description of the Real-time conditions that the Real-time Fuzzy Digital Filter (RTFDF) has perform: the basic results are described in formal sense using the basic definitions considering the concepts develop in others papers referenced in this; finally this work shows a simulation of the RTFDF operation.

Keywords: Fuzzy logic, digital filter, adaptive filter, knowledge base, Real-time.

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1 Introduction

The fuzzy filter system [1], works in loop form and its parameters are adapted in a dynamical form ([2], [4], [6], and [11]) this is based in the error *e(k)* ([5] and [6]) which is the difference between the desired response *d(k)* and the actual signal *y(k)* ([5], [6], [11]).

The criterion is based on the pursuit error *e(k)* permitting this to generate the membership function (the function which approximates the signal *y(k)* to *d(k)*) for to adjust the parameters of the filter which the value of the error *e(k)* should be near to γ [15].

In the fuzzy filtering each rule defined as membership function is limited by a specific sequence given by the response of the system within of the correct operation of the system range and also limited by a distribution function predefined and where the responses of the system are bounded. This concept is described in the follows description:

The fuzzy filter is composed by the next elements considering the concepts studied in [1], [2], [5], [6], [7], [8], [9], [10], and [15]:

1. Rule base, (*IF-THEN*), it contains the quantification of the process by fuzzy logic, realizing an expert linguistic description about how obtain an optimal process filtering, using predefined intervals.
2. Inference Mechanism, this makes the expert decision based on an interpretation and this is applied to know how of what the best way to filter a process by predefined intervals is.
3. Input Fuzzy Inference, this converts the input of the filter into usefulness information to the inference mechanism for to activate and apply the rules.
4. Output Fuzzy Inference, this converts the results or conclusions that the inference mechanism obtains into actualized inputs for the process based in the knowledge base of the system or process that is analyzing.

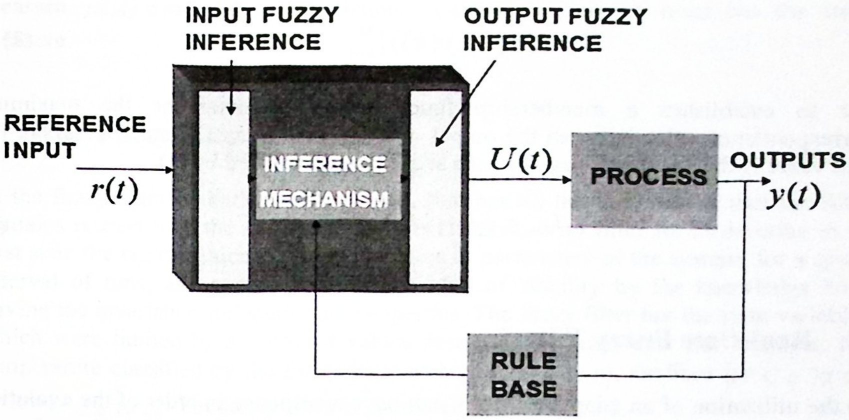

Figure 1. Fuzzy filter process: Description.

2 Membership Functions

DEFINITION: A fuzzy filter is a filter which its error signal ([5] and [15]) is used for to make a membership function ([1], [3], [8], [9], and [10]) required for the adapting algorithm ([2], [4], and [8]).

Here are realized the necessary adjustments for example: regulation of the exciting signals and the filter coefficients using the knowledge base, by this manner we can obtain the minimum previous selection of the filter response with respect to a reference criterion, in a distribution sense.

The most used criterion in the literature as the objective function is the quadratic mean error (QME), described by the second probability moment (to see: [4], [5], and [15]) in a recursive form (1):

$$E_n = \left[\frac{1}{n}\left((n-1)E_{n-1}^2 + e(n)^2\right)\right]^{\frac{1}{2}} \qquad (1)$$

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$$T = \{(d(k), y(k))\}_{k=1}^{N} \quad (2)$$

For to establishes a membership function has to determine the maximum correspondence value between the output *y(k)* and the desired signal *d(k)*, where the best value is the minimum cost for each sequence represented by (3).

$$J_{\min} = \min_{i \to N} \{J(d_i, y_i)\} \quad (3)$$

3 Real-time Fuzzy Filtering

In the utilization of an adaptive filter it can have a response in order of the evolution of an specific process for a kind of excitation, but it requires that the loop for the inputs modifies in a optimum form the system responses, without generate a change in its states, and has to operate in Real-time.

The fuzzy filter classifies its different levels of operation by the membership functions, for to give an specific response by the value of the error function J_{min} to a given input; corresponding a specific response for each kind of input limited by the error function that is also limited by γ. The classification of the system response is realized by the fuzzy filter, for to know the type of the inputs or parameters; permitting to know in an indirect way if the system has invariance, the input is stationary and other properties. It establishes the region where the gains or parameters of the system are, describing the estimation state. The application of the fuzzy filter avoids the initial instability of the error function that is limited by an interval within the system operation.

The classification of the membership functions is realized by limiting the criterion (error function), by metric intervals established in each function. Each membership function is established inside the distribution function of the error criterion and could exits diverse membership functions within it, there are characterized within the distribution function inside the system characteristics. This classification is based in the Lebesgue integral because it can give simple conditions permitting that the multiple integrals be expressed in an iterate form and to describe the system in an ordered form, in almost every point.

For to apply the Lebesgue integral (to see: [5], [13], [14] and [15]) it requires to define in each subset of R a function, and it would be associated with a size or measure $\mu(A) \geq 0$, this function must satisfy some properties: For a segment $A = [a, b]$ the measure is given by its length, $\mu(A) = /b - a/$; If A is the union of the sets $\{A_i : i = \overline{1,n}\}$, disjoints by pairs, and $\mu(A) = \sum_{k=1}^{\infty} \mu(A_k)$, If A is a whole with measure $\mu(A) < \infty$, and its translation $x + A = \{x + y : y \in A\}$ must has the same measure.

4 The Fuzzy Filter Elements

In the fuzzy filter is the knowledge base, that has all the information that the filter contains respect with the system to monitor, pursuit, or to filter for to describe in its first state the region which contains the gains or parameters of the system, for a given interval of time, and can establishes the kind of stability by the knowledge base having the invariance and stationary properties. The fuzzy filter has the state variables which were limited by a range of values described by fuzzy sets (for example, the temperature classified by the fuzzy logic as high$(32^{o}C\ a\ 43^{o}C)$, medium $(17^{o}C\ a\ 32^{o}C)$ or low$(9^{o}C\ a\ 17^{o}C)$. The fuzzy filter is presented in the Figure 2.

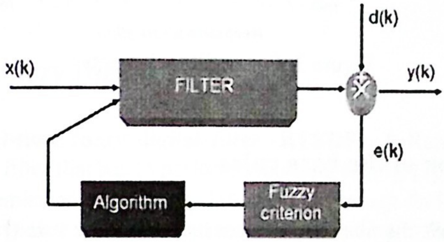

Figure 2. Fuzzy filter

Where $x(k)$ is the linguistic value, $T(k)$ is the set of the representation values that could take $x(k)$ within a limited interval, $d(k)$ is the fuzzy variable of reference, $e(k)$ is the fuzzy value where the set $\{\gamma_i : \gamma_i > 0,\ \forall i \in N\}$ has $inf\{\gamma_i\} \to |\lambda|,\ \lambda > 0$ and the $sup\{\gamma_i\} \to |\lambda|,\ |\lambda| < 1$, where γ^* is small in an infinite form because $y(k)$ should be almost equal to $d(k)$, $e(k)$ is the difference between $y(k)$ and $d(k)$, and the criterion contains the values o membership functions for each case. The elements of the fuzzy filter are shown in Figure 3.

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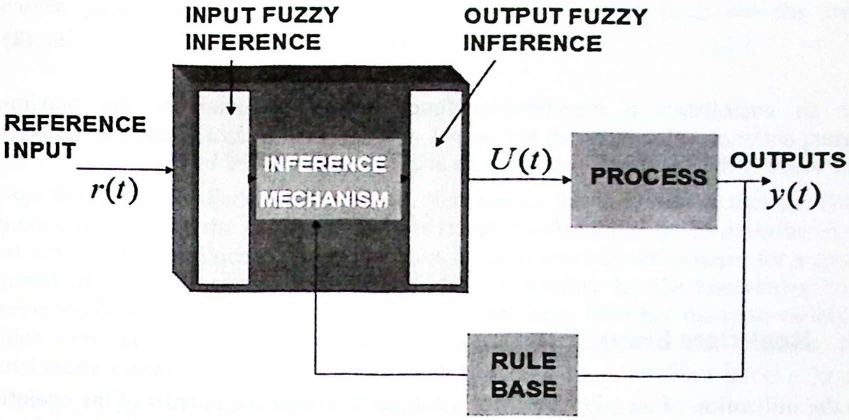

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$$\int m * dm = \int (\underset{k \geq m}{\arg\min\inf} P\{|\hat{a}_k - a| \leq \Delta\} = 1) dP \subseteq [0+\beta, 1-\alpha) \tag{5}$$

Each RTFDF as parameters estimator has a limited error function: where Δ is the limit of the error defined by the variance of the system perturbations, in the fuzzy case this parametric variations within the error limited by the variance represents the membership functions or corresponding criterions for the reference model to each fuzzy variable [11].

Local. Implies the stability of the process trough its parameters $a(k)_i$ actualized at each iteration or at each fuzzy variable and the finalized times $f(k)_i$ of the RTFDF within of the absolute corresponding limits $[ld(k)_{i\ min}, LD(k)_{i\ max})$.

The RTFDF will give a stable local response if the estimation to the set of the SLIT parameters of the filter, being all of them inside of the unit circle for all the k interval considering either convolution with respect to the follows filter. This is:

$$\{a_e(k)\}_i \subseteq [0+\beta, 1-\alpha), \ k = \overline{1,n}. \tag{6}$$

7 Simulation:

For the simulation of the fuzzy filter and to estimate the estates and parameters of the system we were used the Kalman filter [5]. The system was characterized with its operational levels by using fuzzy filtering concepts [11].

The system is described by its estates as:

$$x(k) = a(k)x(k-1) + w(k), \tag{7}$$

And the output is described by:

$$y(k) = x(k) + v(k), \ x(k), \ w(k), \ v(k) \in R^{n \times 1} \tag{8}$$

Where: $x(k)$= is the internal estate of the system, $\{a(k)\}$=are the adjusting parameters, $\{w(k)\}$=are the noises of the system, $y(k)$= is the output of the system, $v(k)$= are the output noises. For to obtain the different levels of operations we must have the mean of the system output $y(k)$, and to get its variance in order to describe the levels of the response (low, medium and high). Figure 4, shows the response levels of the output $y(k)$:

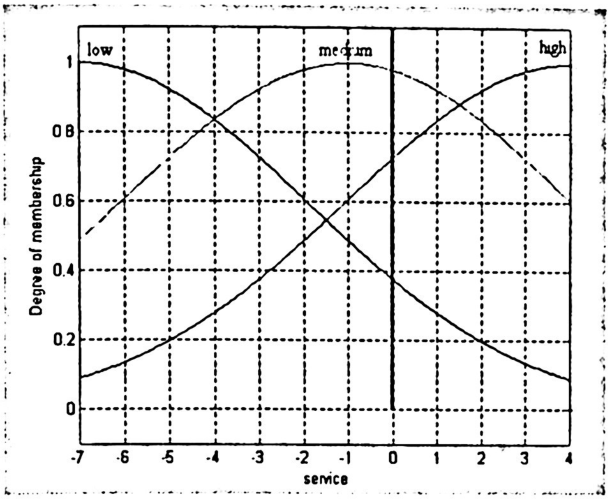

Figure 4. Response levels of $y(k)$

Figure 5, shows the Functional described in (3) with respect to the filter:

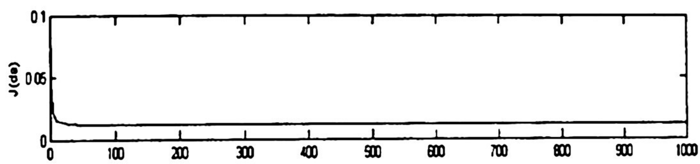

Figure 5. Error functional

Figure 6, shows the Functional Levels in distribution sense, bounded these by the first two probability moments:

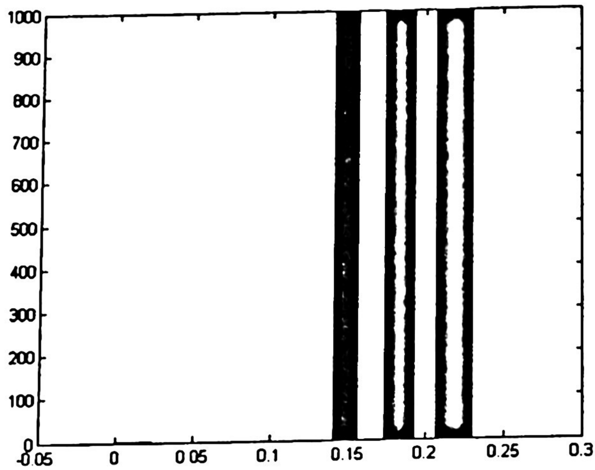

Figure 6. Distribution of error function *J(k)*

Figure 7, shows the Membership Functions classified in probability distribution sense with respect to *J(k)* bounded it by the first two probability moments:

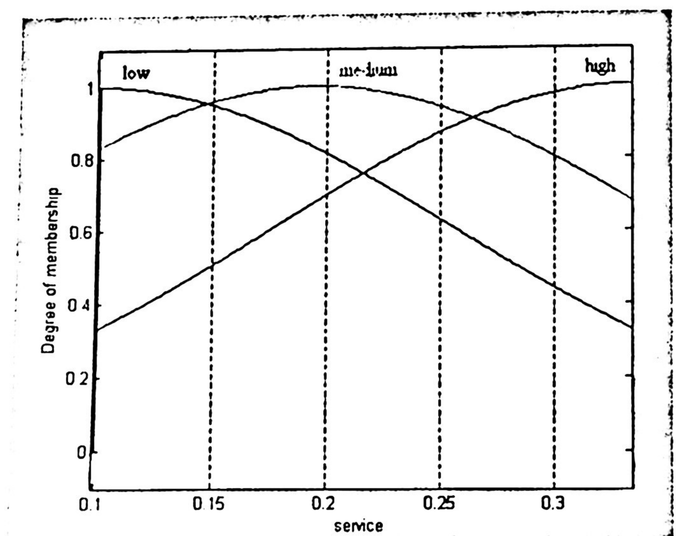

Figure 7. Membership functions *J(k)*

8 Conclusions

This work was presented considering basic concepts of fuzzy logic applied into digital filters with Real-time systems, which made origin to the basic concept of real time fuzzy digital filters, described as RTFDF. We were presented here the basic operation characteristics and meaning of this kind of filters in order to know how the signals was classified by the filter, we resolved how to construct and characterize the Membership Function and the Knowledge Base in a probabilistic manner, making a description of the real time conditions that the RTFDF was performed, and finally this work showed an illustrative simulation of the RTFDF operation.

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